Statistics

Winter 2022

Lecture 6

Addition Rule:

Keyword OR

Single Action event

$P(A \text{ or } B) = P(A) + P(B) - \underbrace{P(A \text{ and } B)}_{\text{Both}}$

ex: $P(A) = .7$, $P(B) = .5$, $P(A \text{ and } B) = .3$

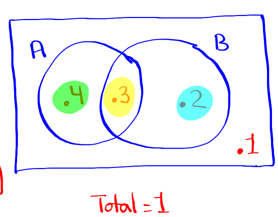
$P(\bar{A}) = 1 - P(A) = \boxed{.3}$

$P(\bar{B}) = 1 - P(B) = \boxed{.5}$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$= .7 + .5 - .3$

$= \boxed{.9}$



$P(A \text{ only}) = .7 - .3 = .4$

$P(B \text{ only}) = .5 - .3 = .2$

$1 - [.4 + .3 + .2] = 1 - .9 = \boxed{.1}$

$P(\overline{A \text{ or } B}) = .1$

De Morgan's Law:

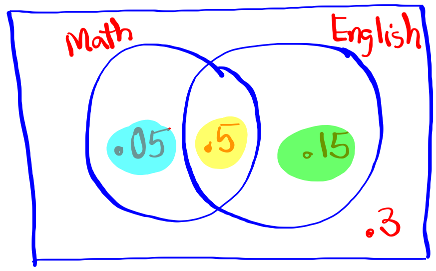
$P(\bar{A} \text{ and } \bar{B}) = P(\overline{A \text{ or } B}) = \boxed{.1}$

$P(\bar{A} \text{ or } \bar{B}) = P(\overline{A \text{ and } B}) = 1 - .3 = \boxed{.7}$

P(Math) = .55

Construct Venn Diagram

P(English) = .65



P(Math and English) = .5

P(Math only) = .55 − .5 = $\boxed{.05}$

P(English only) = .65 − .5 = $\boxed{.15}$

1 − [.05 + .5 + .15] = 1 − .7 = $\boxed{.3}$

Total = 1 ✓

$P(\overline{\text{Math}} \text{ and } \overline{\text{English}}) = P(\overline{\text{Math or English}}) = \boxed{.3}$

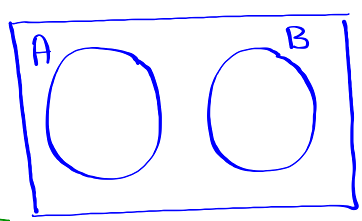
De Morgan's Law

$P(\overline{\text{Math}} \text{ OR } \overline{\text{English}}) = P(\overline{\text{Math and English}}) = \boxed{.5}$

Mutually Exclusive Events (Disjoint events)

P(A and B) = 0

No overlap

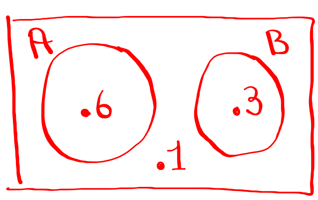


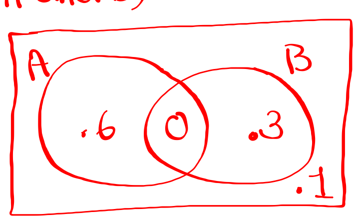
Ex: P(A) = .6

P(B) = .3

A and B are M.E.E.

P(A and B) = 0





$P(\overline{A}) = 1 - .6 = \boxed{.4}$

$P(\overline{B}) = 1 - .3 = \boxed{.7}$

P(A or B) = .6 + .3 − 0

$= \boxed{.9}$

$P(\text{get grade A}) = .35$

$P(\text{get grade B}) = .45$

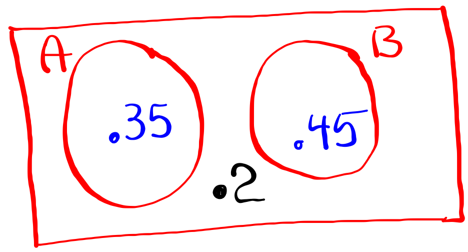
Construct Venn Diagram

A and B are disjointed events

M.E.E.

No overlap

$P(\text{get grade A or B})$

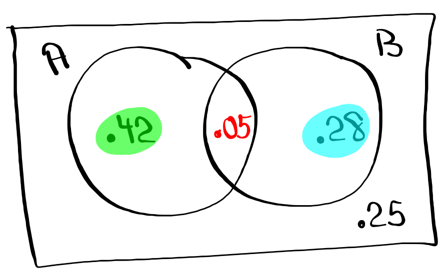


$P(\text{grade A or B}) =$

$.35 + .45 = \boxed{.8}$

Total = 1

Complete the Venn Diagram below



Total = 1

$1 - [.42 + .28 + .25] = .05$

$P(A) = .42 + .05 = \boxed{.47}$

$P(B) = .28 + .05 = \boxed{.33}$

$P(\text{A only or B only}) = .42 + .28 = \boxed{.7}$

$P(\overline{A} \text{ and } \overline{B}) = P(\overline{A \text{ or } B}) = \boxed{.25}$

De Morgan's Law

$P(\overline{A} \text{ or } \overline{B}) = P(\overline{A \text{ and } B}) = 1 - .05 = \boxed{.95}$

Multiplication Rule

Keyword AND

multiple action event

Type I: Independent events

outcome of one event does not change the prob. of next event.

P(New Born is boy) = .5

Toss a fair coin

P(land tails) = .5

Multiple-choice question

4 choices, but one correct choice in every question

P(guess correct) = $\frac{1}{4}$ per question.

If A and B are independent events,

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

A happens first, then B happens.

Ex: A fair coin is tossed twice.

P(Tails) = .5

P(Two Tails) = P(First Tail) · P(Second Tail)

$= (.5)(.5) = \boxed{.25}$

List of all outcomes

TT TH HT HH

Consider a loaded coin such that

$P(T) = .7$, $P(H) = .3$

toss this coin twice

$P(TT) = (.7)(.7) = \boxed{.49}$

$P(HH) = (.3)(.3) = \boxed{.09}$

$P(1T \ \& \ 1H) = P(TH \text{ or } HT)$

$= (.7)(.3) + (.3)(.7) = \boxed{.42}$

You are making random guesses on a quiz with 4 questions.

Each question has 3 choices but only one correct choice.

$P(\text{Correct}) = \frac{1}{3}$ $\qquad P(\overline{\text{Correct}}) = \frac{2}{3}$ Per question.

$P(CCCC) = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{81}$

$P(\bar{C}\bar{C}\bar{C}\bar{C}) = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \boxed{\frac{16}{81}}$

January 13, 2022

Ex:

$P(A) = .4$ $\quad P(B) = .3$ $\quad$ A and B are independent events

$P(\bar{A}) = 1 - P(A) = \boxed{.6}$

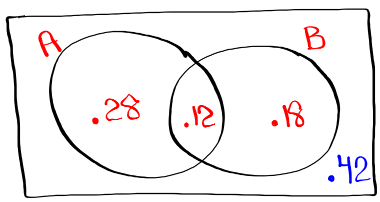
$P(\bar{B}) = 1 - P(B) = \boxed{.7}$

$P(A \text{ and } B) = P(A) \cdot P(B) = (.4)(.3) = \boxed{.12}$

$P(A \text{ or } B) =$ Addition Rule

$= P(A) + P(B) - P(A \text{ and } B)$

$= .4 + .3 - .12 = \boxed{.58}$



Total = 1

$P(A) = .7$, $P(B) = .2$ Find $P(A \text{ and } B)$

a) if A & B are M.E.E.

Disjoint events

$P(A \text{ and } B) = \boxed{0}$

b) if A & B are independent events

$P(A \text{ and } B) = P(A) \cdot P(B)$

$= (.7)(.2)$

$= \boxed{.14}$

Odds

odds in favor of event E are $a:b$.

odds against event E are $b:a$.

odds in favor of event E are $a:b$.

E happens

E does not happen

I tossed a coin 20 times.

I got 7 Tails & 13 Heads.

odds in favor of landing tails are $7:13$.

odds against landing tails " $13:7$.

A deck of playing cards with 40 cards has 15 red, 8 face cards, and 3 Aces.

Find odds in favor of drawing

a) a red card

$\#\text{Red} : \#\overline{\text{Red}}$

$15 : 25$ $\boxed{3:5}$

Simplify

b) a face card

$\#\text{face} : \#\overline{\text{face}}$

$8 : 32 \Rightarrow \boxed{1:4}$

c) an Ace

$\#\text{Ace} : \#\text{Aces}$

$\boxed{3 : 37}$

If odds in favor of event E are a:b,

then $P(E) = \frac{a}{a+b}$ & $P(\overline{E}) = \frac{b}{a+b}$

Ex: Suppose the odds in favor of event E are 4:21.

1) odds against event E

$\boxed{21:4}$

2) $P(E) = \frac{4}{4+21} = \boxed{\frac{4}{25}}$

3) $P(\overline{E}) = \frac{21}{4+21} = \frac{21}{25}$

How to find odds using Probability:

If P(E) is given,

odds in favor of E are $P(E):P(\overline{E})$

Always Simplify

P(Lakers win championship this year) = .15

$P(W) = .15$, $P(\overline{W}) = .85$

Odds in favor $P(W):P(\overline{W})$

.15 : .85 ⇒ 3:17

bet

Net earning

.15 [÷] .85 [MATH] [1:▸Frac] [Enter] $\frac{3}{17}$

If you bet \$3 on lakers,

and they win the championship,

you collect \$20 (Net Earning \$17)

Suppose P(win a hand) = 2.5%

$P(W) = .025$

$P(\overline{W}) = 1 - .025 = .975$

odds to win ⇒ $.025 : .975$

Simplify

$1 : 39$

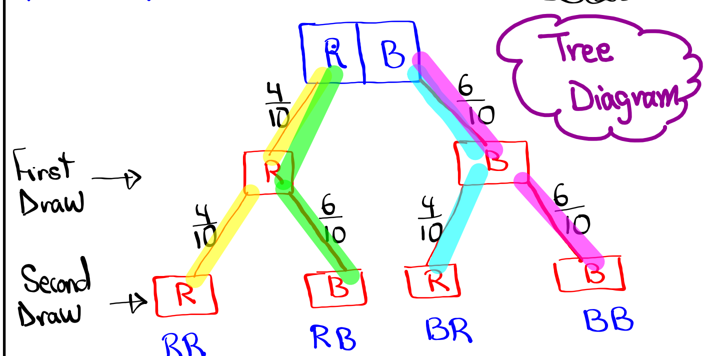
\$1 bet ⇒ \$39 Net

How much should I bet to net \$780?

$\frac{\$1 \text{ bet}}{\$39 \text{ Net}} \qquad \frac{\$x \text{ bet}}{\$780} \qquad \frac{1}{39} = \frac{x}{780}$

$x = 20$ \$20

A box has 4 Red and 6 Blue Balls.
Randomly take two balls with replacement.



$P(RR) = \frac{4}{10} \cdot \frac{4}{10} = \frac{16}{100} = \boxed{.16}$ ✓

$P(RB) = \frac{4}{10} \cdot \frac{6}{10} = \frac{24}{100} = \boxed{.24}$ ✓

$P(BR) = \frac{6}{10} \cdot \frac{4}{10} = \frac{24}{100} = \boxed{.24}$ ✓

$P(BB) = \frac{6}{10} \cdot \frac{6}{10} = \frac{36}{100} = \boxed{.36}$ ✓

Total = 1

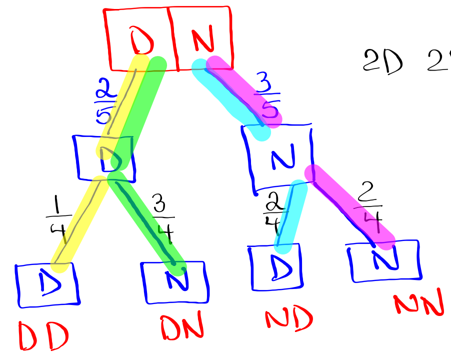
A box has 2 Dimes & 3 Nickels.
Randomly take 2 coins, No replacement.

1D 3N

2D 2N

First Draw

Second Draw



$P(DD) = \frac{2}{5} \cdot \frac{1}{4} = \frac{2}{20} = \boxed{.1}$

$P(DN) = \frac{2}{5} \cdot \frac{3}{4} = \frac{6}{20} = \boxed{.3}$

$P(ND) = \frac{3}{5} \cdot \frac{2}{4} = \frac{6}{20} = \boxed{.3}$

$P(NN) = \frac{3}{5} \cdot \frac{2}{4} = \frac{6}{20} = \boxed{.3}$

Total = 1

There are 3W and 5M.
I need to select 2 people

✓WW
✓W M
✓M W
M M

} Sample Space

$P(2 \text{ Women}) = \frac{3}{8} \cdot \frac{2}{7} = \boxed{\frac{3}{28}}$

$P(2 \text{ Men}) = \frac{5}{8} \cdot \frac{4}{7} = \boxed{\frac{5}{14}}$

$P(\text{at least } 1 \text{ Woman}) = 1 - P(\text{No Woman})$
$= 1 - P(MM)$
$= 1 - \frac{5}{14} = \boxed{\frac{9}{14}}$

Prob. with at least 1

$\boxed{P(\text{at least } 1) = 1 - P(\text{None})}$

Standard deck of Playing cards

52 Cards, 4 Aces.

Draw 2 Cards. No replacement.

$A \; A$

$A \; \bar{A}$

$\bar{A} \; A$

$\bar{A} \; \bar{A}$

$P(2 \text{ Aces}) = \frac{4}{52} \cdot \frac{3}{51} = \frac{1}{221}$

$4 \div 52 \cdot 3 \div 51$ Math 1 ▸Frac Enter

$P(\text{No Aces}) = \frac{48}{52} \cdot \frac{47}{52} = \frac{188}{221}$

$P(\text{at least 1 Ace}) = 1 - P(\text{No Aces})$

$= 1 - \frac{188}{221} = \boxed{\frac{33}{221}}$

Selecting without replacement is an example of dependent events.

Prob. changes after first selection.

General Multiplication Rule

$P(A \text{ and } B) = P(A) \cdot P(B|A)$

A happens, then B happens

Given

4 Red & 8 Blue

Draw 2 Balls, No replacement

$P(\text{two Reds}) = \frac{4}{12} \cdot \frac{3}{11} = \frac{1}{11}$

$P(\text{Two Blues}) = \frac{8}{12} \cdot \frac{7}{11} = \frac{14}{33}$

$P(\text{at least 1 Red}) = 1 - P(\text{No Red})$

$= 1 - \frac{14}{33} = \frac{19}{33}$

$P(\text{at least 1 Blue}) = 1 - P(\text{No Blue}) = 1 - \frac{1}{11} = \frac{10}{11}$

Live QZ 1:

Consider the Sample below

15 18 20 10 16

25 20 30 24 16

Find

$\bar{x} = 19.4$

$S = 5.758$

S^2 (Reduced Fraction) $= \frac{1492}{45}$